FA·4.
WORK SHEET

Ch. 9
RATIONAL NUMBERS

(i) $-7 \times (-16) = 112$; $8 \times 14 = 112$

Since, $-7 \times (-16) = 8 \times 14 = 112$. So, $\frac{-7}{8} = \frac{14}{-16}$.

Hence, $\frac{-7}{8}$ and $\frac{14}{-16}$ are equivalent rational numbers.

(ii) $5 \times 18 = 90$; $(-3) \times (-15) = 45$

Since $5 \times 18 \neq (-3) \times (-15)$.

So, $\frac{5}{-3} \neq \frac{-15}{18}$.

Hence, $\frac{5}{-3}$ is not equivalent to $\frac{-15}{18}$.

Example 7: *Show that* $\frac{-16}{20}$ *and* $\frac{4}{-5}$ *are equivalent rational numbers.*

Since $-16 \times -5 = 80$ and $20 \times 4 = 80$.

$\therefore (-16) \times (-5) = 20 \times 4$

Hence, $\frac{-16}{20}$ and $\frac{4}{-5}$ are equivalent rational numbers.

Example 8: *Find the value of x, such that.*

(i) $\frac{-8}{20} = \frac{x}{5}$ *(ii)* $\frac{x}{13} = \frac{-8}{39}$

(i) $\frac{-8}{20} = \frac{x}{5}$

$\Rightarrow -8 \times 5 = x \times 20$

$\Rightarrow x = \frac{-8 \times 5}{20} = -2$.

(ii) $\frac{x}{13} = \frac{-8}{39}$

$\Rightarrow x \times 39 = -8 \times 13$

$\Rightarrow x = \frac{-8 \times 13}{39} = \frac{-8}{3}$.

Exercise 9.1

RATIONAL NOS.

1. What do you understand by a rational number? Write any five rational numbers.
2. **Write the numerator and the denominator of each of the following rational numbers.**

   (i) $\frac{-148}{-249}$ (ii) $\frac{7}{13}$ (iii) $\frac{-3}{13}$ (iv) $\frac{2}{-29}$

3. **Do as directed.**

   (i) Write 0 as a rational number.

   (ii) Write any three natural numbers as rational numbers.

   (iii) Write any three negative integers as rational numbers.

4. **Are the following rational numbers positive or negative?**

(i) $\frac{-4}{5}$ (ii) $\frac{-5000}{-11000}$ (iii) $\frac{-3}{-21}$ (iv) $\frac{3}{-7}$

5. **Fill in the boxes.**

(i) $\frac{3}{-11}=\frac{21}{\square}=\frac{\square}{-99}$ (ii) $\frac{5}{13}=\frac{35}{\square}=\frac{\square}{182}$

(iii) $\frac{-3}{-4}=\frac{\square}{-20}=\frac{-21}{\square}$ (iv) $\frac{12}{60}=\frac{\square}{30}=\frac{4}{\square}$

6. **Write each of the following rational numbers with a positive denominator.**

(i) $\frac{-147}{-189}$ (ii) $\frac{8}{-21}$ (iii) $\frac{-13}{-27}$ (iv) $\frac{39}{-59}$

7. **Write each of the following rational numbers with a positive numerator.**

(i) $\frac{-8}{13}$ (ii) $\frac{-18}{-29}$ (iii) $\frac{-47}{-35}$ (iv) $\frac{-47}{38}$

8. **Express $\frac{-3}{7}$ as a rational number with denominator.**

(i) -35 (ii) 70 (iii) 63 (iv) -21

9. **Express $\frac{8}{-13}$ as a rational number with numerator.**

(i) 16 (ii) -24 (iii) 40 (iv) -64

10. **Express $\frac{-72}{360}$ as a rational number with denominator.**

(i) -10 (ii) -40 (iii) -20 (iv) 60

11. **Express $\frac{420}{-720}$ as a rational number with numerator.**

(i) -105 (ii) -35 (iii) 70 (iv) -28

12. **Write the following rational numbers in standard form.**

(i) $\frac{39}{-78}$ (ii) $\frac{38}{-56}$ (iii) $\frac{-24}{-72}$ (iv) $\frac{8}{32}$

(v) $\frac{234}{-468}$ (vi) $\frac{-121}{-242}$

13. **Check if the following pairs of rational numbers are equivalent.**

(i) $\frac{-8}{-13}, \frac{-5}{16}$ (ii) $\frac{5}{-7}, \frac{21}{-15}$ (iii) $\frac{-3}{8}, \frac{-6}{16}$ (iv) $\frac{-4}{-12}, \frac{-6}{-18}$

14. **Find x such that.**

(i) $\frac{-21}{8}=\frac{x}{16}$ (ii) $\frac{-49}{x}=-7$ (iii) $\frac{x}{112}=-5$ (iv) $\frac{13}{-17}=\frac{26}{x}$

2. **In each of the following pairs of rational numbers, find the greater one.**

(i) $-\frac{8}{7}, 0$ (ii) $\frac{-21}{35}, \frac{7}{8}$ (iii) $\frac{4}{7}, 0$ (iv) $\frac{-5}{13}, \frac{8}{-3}$

(v) $\frac{-18}{-35}, 0$ (vi) $\frac{-5}{17}, \frac{8}{15}$ (vii) $\frac{-7}{8}, \frac{5}{-12}$ (viii) $\frac{7}{19}, \frac{-5}{38}$

3. **In each of the following pairs of rational numbers, find the smaller one.**

(i) $\frac{-6}{15}, \frac{4}{-9}$ (ii) $\frac{-8}{-21}, \frac{13}{14}$ (iii) $\frac{5}{-6}, \frac{-7}{21}$ (iv) $\frac{-9}{14}, \frac{7}{-16}$

(v) $\frac{14}{13}, \frac{-3}{-26}$ (vi) $\frac{-3}{-8}, \frac{4}{7}$

4. **Fill in the $\square$ with the correct symbol out of >, = or <.**

(i) $\frac{-3}{8} \square \frac{6}{-16}$ (ii) $\frac{-4}{7} \square \frac{-3}{14}$ (iii) $\frac{-2}{5} \square \frac{5}{-7}$ (iv) $\frac{-3}{7} \square \frac{4}{-7}$

5. **Arrange in ascending order.**

(i) $\frac{3}{-5}, \frac{4}{7}, \frac{-3}{14}, \frac{7}{10}$ (ii) $\frac{-4}{9}, \frac{-3}{7}, \frac{8}{15}, \frac{13}{-21}$

(iii) $\frac{-2}{5}, \frac{-8}{7}, \frac{3}{-35}, \frac{-11}{45}$ (iv) $\frac{-8}{11}, \frac{-19}{22}, \frac{105}{-154}, \frac{-119}{121}$

6. **Arrange in descending order.**

(i) $\frac{5}{8}, \frac{13}{-16}, \frac{-15}{12}, \frac{1}{6}$ (ii) $\frac{1}{-5}, \frac{-3}{10}, \frac{17}{15}, \frac{13}{20}$

(iii) $\frac{2}{-3}, \frac{-4}{9}, \frac{8}{27}, \frac{7}{-12}$ (iv) $\frac{3}{-7}, \frac{8}{13}, \frac{-4}{39}, \frac{-15}{91}$

Addition of Rational Numbers

The addition of rational numbers is carried out in the same way as that of addition of fractions. If two rational numbers are to be added, we should first convert each of them into rational numbers with a positive denominator.

RATIONAL NUMBERS WITH SAME DENOMINATORS

To add two rational numbers with same denominators, we simply add their numerators and divide the sum by the common denominator.

In general if $\frac{p}{q}$ and $\frac{r}{q}$ are any two rational numbers, then $\frac{p}{q} + \frac{r}{q} = \frac{p+r}{q}$.

Quick Note

Reduce the sum into the standard form (simplest form) wherever possible.

RATIONAL NUMBERS WITH DIFFERENT DENOMINATORS

To add rational numbers with different denominators, we follow the steps given below:

1. In case the denominators of the given rational numbers are negative, change them into positive denominators.
2. Find the L.C.M. of the denominators.
3. Express each rational number with the L.C.M. as the common denominator.
4. Add the rational numbers.

Solved Examples

Example 1: *Add* $\frac{5}{9}$ *and* $\frac{8}{-9}$.

Since, $\frac{8}{-9} = \frac{8 \times (-1)}{(-9) \times (-1)} = \frac{-8}{9}$.

So, $\frac{5}{9} + \frac{8}{-9} = \frac{5}{9} + \frac{(-8)}{9} = \frac{5+(-8)}{9} = \frac{5-8}{9} = \frac{-3}{9} = \mathbf{\frac{-1}{3}}$.

Example 2: *Add* $\frac{6}{11}$ *and* $\frac{13}{11}$.

We have, $\frac{6}{11} + \frac{13}{11} = \frac{6+13}{11} = \mathbf{\frac{19}{11}}$.

Example 3: *Add* $\frac{5}{-8}$ and $\frac{-5}{12}$.

The denominator of $\frac{5}{-8}$ is negative. Multiply the numerator and the denominator with (−1) to get the positive denominator,

$\frac{5}{-8} = \frac{5 \times (-1)}{-8 \times (-1)} = \frac{-5}{8}$

The L.C.M. of 8 and 12 is 24.

Now, $\frac{5}{-8} = \frac{-5}{8} = \frac{-5 \times 3}{8 \times 3} = \frac{-15}{24}$, and $\frac{-5}{12} = \frac{-5 \times 2}{12 \times 2} = \frac{-10}{24}$.

$\therefore \frac{5}{-8} + \frac{-5}{12} = \frac{-15}{24} + \frac{-10}{24} = \frac{-15+(-10)}{24} = \frac{-15-10}{24} = \mathbf{\frac{-25}{24}}$.

Example 4: *Add* $\frac{4}{5}$ and $\frac{-3}{4}$.

The denominators of the given rational numbers are positive.

The L.C.M. of 5 and 4 is 20.

Now, $\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20}$, and $\frac{-3}{4} = \frac{-3 \times 5}{4 \times 5} = \frac{-15}{20}$.

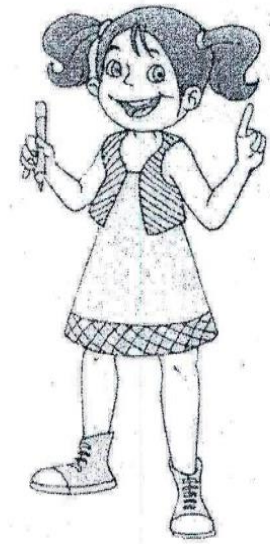
$\therefore \frac{4}{5} + \frac{(-3)}{4} = \frac{16}{20} + \frac{(-15)}{20} = \frac{16-15}{20} = \mathbf{\frac{1}{20}}$.

Example 5: *Add* $\frac{-21}{6}$ and $\frac{-19}{5}$ *and write the sum as a mixed fraction.*

L.C.M. of 6 and 5 is 30.

Now, $\frac{-21}{6} = \frac{-21 \times 5}{6 \times 5} = \frac{-105}{30}$, and $\frac{-19}{5} = \frac{-19 \times 6}{5 \times 6} = \frac{-114}{30}$

$\therefore \frac{-21}{6} + \frac{-19}{5} = \frac{-105}{30} + \frac{-114}{30} = \frac{-105+(-114)}{30} = \frac{-219}{30} = -7\mathbf{\frac{9}{30}}$.



Example 6: *Find the sum of the three rational numbers:* $\frac{-5}{8}, \frac{-7}{12}$ and $\frac{11}{16}$.

The denominators of the given rational numbers are all positive. The L.C.M. of 8, 12 and 16 is 48.

Now, $\frac{-5}{8} = \frac{-5 \times 6}{8 \times 6} = \frac{-30}{48}$, $\frac{-7}{12} = \frac{-7 \times 4}{12 \times 4} = \frac{-28}{48}$, and $\frac{11}{16} = \frac{11 \times 3}{16 \times 3} = \frac{33}{48}$.

$\therefore \quad \frac{-5}{8} + \frac{-7}{12} + \frac{11}{16} = \frac{-30}{48} + \frac{-28}{48} + \frac{33}{48} = \frac{-30-28+33}{48} = \frac{-25}{48}$.

Exercise 9.3

1. **Add the following rational numbers.**

(i) $\frac{4}{7}$ and $\frac{4}{7}$ (ii) $\frac{-4}{5}$ and $\frac{2}{5}$ (iii) $\frac{-5}{11}$ and $\frac{9}{-11}$ (iv) $\frac{5}{-8}$ and $\frac{1}{8}$

2. **Add the following rational numbers.**

(i) $\frac{19}{8}$ and $\frac{7}{3}$ (ii) $\frac{33}{18}$ and $\frac{17}{26}$ (iii) $\frac{-7}{27}$ and $\frac{5}{18}$ (iv) $\frac{-9}{10}$ and $\frac{22}{15}$

3. **Evaluate.**

(i) $\frac{11}{-12} + \frac{3}{-8} + \frac{1}{4}$ (ii) $\frac{-12}{7} + \frac{5}{7} + \frac{-4}{7}$ (iii) $\frac{-3}{5} + \frac{8}{5} + \frac{-2}{5}$ (iv) $\frac{13}{27} + \frac{11}{3} + 4$

(v) $\frac{-15}{27} + \frac{-8}{3}$ (vi) $1 + \frac{-8}{13} + 5$ (vii) $1 + \frac{-7}{12} + \frac{5}{6}$ (viii) $\frac{-4}{3} + \frac{7}{18} + \frac{4}{21}$

4. **Simplify.**

(i) $\frac{-9}{8} + \frac{5}{-16}$ (ii) $\frac{-7}{20} + \frac{14}{-15} + \frac{1}{10}$ (iii) $-1 + \frac{7}{-9} + \frac{11}{12}$ (iv) $\frac{-8}{15} + \frac{2}{-3}$

5. **Add and write the sum as a mixed fraction.**

(i) $\frac{-23}{8}$ and $\frac{1}{4}$ (ii) $\frac{18}{-11}$ and $\frac{-6}{5}$ (iii) $\frac{-15}{7}$ and $\frac{23}{-14}$ (iv) $\frac{-101}{25}$ and $-\frac{11}{15}$

(v) $\frac{14}{5}$ and $\frac{8}{15}$ (vi) $\frac{31}{7}$ and $\frac{15}{8}$ (vii) $\frac{121}{-15}$ and $\frac{-31}{5}$ (viii) $\frac{-13}{6}$ and $\frac{-13}{7}$

Properties of Addition of Rational Numbers

ASSOCIATIVE PROPERTY

The sum of three or more rational numbers remains the same irrespective of the way in which they are grouped.

In general, *if* $\frac{p}{q}, \frac{r}{s}$ *and* $\frac{m}{n}$ *be any three rational numbers, then* $\frac{p}{q} + \left(\frac{r}{s} + \frac{m}{n}\right) = \left(\frac{p}{q} + \frac{r}{s}\right) + \frac{m}{n}$

For example; let us find the sum of $\frac{1}{2}, \frac{3}{4}$ and $\frac{-1}{6}$.

$$\frac{1}{2} + \left(\frac{3}{4} + \frac{-1}{6}\right) = \frac{1}{2} + \frac{9-2}{12} = \frac{1}{2} + \frac{7}{12} = \frac{6+7}{12} = \frac{13}{12}$$

ZERO PROPERTY

If we subtract 0 from any non-zero rational number, the result is the non-zero rational number itself.

In general; *if* $\frac{p}{q}$ *is a non-zero rational number, then* $\frac{p}{q}-0=\frac{p}{q}$.

The commutative and associative properties do not hold for subtraction of rational numbers.

Solved Examples

Example 1: *Subtract* $\frac{-5}{3}$ *from* $\frac{-1}{4}$.

The additive inverse of $\frac{-5}{3}$ is $\frac{5}{3}$.

$\therefore\ \frac{-1}{4}-\left(\frac{-5}{3}\right)=\frac{-1}{4}+\frac{5}{3}=\frac{-3+20}{12}=\frac{17}{12}$.

Example 2: *What number should be added to* $\frac{-3}{5}$*, so as to get* $\frac{15}{8}$*?*

The given number is $\frac{-3}{5}$.

The required number is $\frac{15}{8}-\left(\frac{-3}{5}\right)=\frac{15}{8}+\frac{3}{5}=\frac{75+24}{40}=\frac{99}{40}$.

Example 3: *The sum of two rational numbers is – 4. If one number is* $\frac{-9}{7}$*, find the other.*

The sum of the two rational numbers is – 4 and one number is $\frac{-9}{7}$.

$\therefore$ The other number $=-4-\left(\frac{-9}{7}\right)=\frac{-4}{1}+\frac{9}{7}=\frac{-28+9}{7}=\frac{-19}{7}$.

> **Quick Note**
>
> Subtracting a non-zero rational number $\left(\frac{p}{q}\right)$ from 0, we get the additive inverse of the non-zero rational number, *i.e.*, $0-\frac{p}{q}=-\frac{p}{q}$.

Exercise 9.5

1. Subtract.

(i) $\frac{6}{7}-\frac{3}{7}$ (ii) $\frac{-8}{9}-\frac{-13}{18}$ (iii) $0-\frac{-9}{13}$ (iv) $\frac{3}{8}-\frac{7}{8}$

(v) $\frac{-3}{8}-(-5)$ (vi) $\frac{13}{26}-\frac{-14}{28}$ (vii) $\frac{0}{3}-\frac{-0}{5}$ (viii) $\frac{1}{3}-\frac{-5}{6}$

2. Subtract.

(i) $\frac{17}{51}$ from $\frac{-8}{17}$ (ii) $\frac{3}{-7}$ from $\frac{-19}{21}$ (iii) $\frac{13}{17}$ from $\frac{4}{15}$ (iv) $\frac{-17}{27}$ from 0

(v) 0 from $\frac{-8}{13}$ (vi) $\frac{3}{8}$ from $\frac{4}{7}$

3. Fill in the blanks.

(i) $\frac{-4}{13}-\frac{3}{26}=$ ..... (ii) ...... + $\frac{15}{23}=4$ (iii) $\frac{-7}{9}+$ ..... = 3 (iv) $\frac{-5}{14}+$ ...... = – 1

5. **Verify the associative property of addition for the following rational numbers.**

(i) $\left(\frac{-7}{9}+\frac{3}{8}\right)+\frac{-5}{6}=\frac{-7}{9}+\left(\frac{3}{8}+\frac{-5}{6}\right)$

(ii) $\left(\frac{-3}{11}+\frac{-7}{8}\right)+\frac{13}{-5}=\frac{-3}{11}+\left(\frac{-7}{8}+\frac{13}{-5}\right)$

(iii) $-3+\left(\frac{2}{9}+\frac{-3}{4}\right)=\left(-3+\frac{2}{9}\right)+\frac{-3}{4}$

(iv) $\left(\frac{3}{4}+\frac{-2}{5}\right)+\frac{-7}{10}=\frac{3}{4}+\left(\frac{-2}{5}+\frac{-7}{10}\right)$

6. **Verify that $x+(y+z)=(x+y)+z$ for.**

(i) $x=\frac{-8}{13}, y=\frac{21}{26}, z=\frac{-4}{39}$

(ii) $x=\frac{7}{11}, y=\frac{3}{-22}, z=\frac{8}{9}$

(iii) $x=-3, y=\frac{-5}{7}, z=\frac{6}{-7}$

(iv) $x=\frac{-3}{5}, y=\frac{-8}{9}, z=\frac{-4}{-7}$

7. **Evaluate the following.**

(i) $\frac{5}{12}+\frac{-1}{3}-4+\frac{2}{9}+\frac{-5}{6}+\frac{-7}{9}$

(ii) $\frac{6}{5}+\frac{-8}{15}+\frac{3}{10}+\frac{-11}{5}$

(iii) $3+\frac{8}{15}+\frac{-1}{5}+\frac{3}{10}+\frac{7}{5}+\frac{-11}{10}$

(iv) $\frac{2}{3}+\frac{4}{9}+\frac{-5}{6}+\frac{-8}{12}$

8. **Find the additive inverse or negative of each of the following.**

(i) -17 (ii) $\frac{21}{9}$ (iii) $\frac{2}{3}$ (iv) $\frac{-18}{7}$

(v) $\frac{-16}{-5}$ (vi) $\frac{15}{-7}$ (vii) $\frac{-2}{9}$ (viii) $\frac{-13}{-38}$

(ix) $\frac{-5}{-6}$ (x) $\frac{23}{-6}$ (xi) 0 (xii) $\frac{0}{-8}$

Subtraction of Rational Numbers

We know that subtraction is the inverse of addition, that means in subtraction we add the additive inverse of the number.

Thus if $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers, then subtracting $\frac{r}{s}$ from $\frac{p}{q}$ means adding additive inverse of $\frac{r}{s}$ to $\frac{p}{q}$, i.e., $\frac{p}{q}-\frac{r}{s}=\frac{p}{q}+\left(\frac{-r}{s}\right)$.

Thus, subtracting a rational number means adding its additive inverse.

$\therefore \quad \frac{p}{q}-\frac{r}{s}=\frac{p}{q}+\left(\text{additive inverse of } \frac{r}{s}\right)$.

Properties of Subtraction

CLOSURE PROPERTY

The difference of two rational numbers is always a rational number.

In general; *if $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers, then $\left(\frac{p}{q}-\frac{r}{s}\right)$ is also a rational number.*