INTERNATIONAL INDIAN SCHOOL, RIYADH

FA3 WORKSHEET

Ch. 6. TRIANGLES & Its PROP.

Example 9: *If the sides of a triangle are produced in order, show that the sum of the exterior angles so formed is 360°.* its Properties.

In ΔABC in figure, we have

$\angle 1 + \angle 4 = 180^\circ$ ...(i) [Linear pair property]

$\angle 2 + \angle 5 = 180^\circ$ ...(ii) [Linear pair property]

$\angle 3 + \angle 6 = 180^\circ$ ...(iii) [Linear pair property]

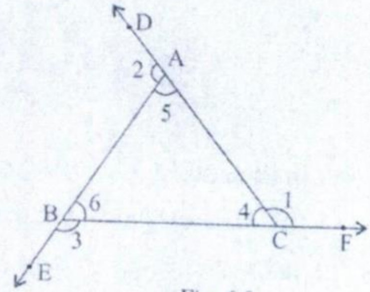


Fig. 6.8

Adding (i), (ii) and (iii), we get

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 180^\circ + 180^\circ + 180^\circ$

or $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 540^\circ$

or $(\angle 1 + \angle 2 + \angle 3) + (\angle 4 + \angle 5 + \angle 6) = 540^\circ$

or $(\angle 1 + \angle 2 + \angle 3) + 180^\circ = 540^\circ$ $[\because \angle 4 + \angle 5 + \angle 6 = 180^\circ]$

or $\angle 1 + \angle 2 + \angle 3 = 540^\circ - 180^\circ$

or $\angle 1 + \angle 2 + \angle 3 = 360^\circ$

Hence, the sum of the exterior angles of a triangle is **360°**.

Exercise 6.1

1. Fill in the blanks.

(i) Triangle has ________ elements.

(ii) A triangle has ________ vertices, ________ sides and ________ angles.

(iii) All the points on the triangle as well as in the interior of the triangle form ________ region.

(iv) If one side of a triangle is produced, the exterior angle so formed is equal to the sum of the ________.

(v) All the angles of an equilateral triangle are ________.

(vi) A triangle has ________ medians and ________ altitudes.

(vii) A ________ connects a vertex of a triangle to the midpoint of the opposite side.

2. Look at each figure below and state whether the triangle is obtuse, acute or right angled.

(i)

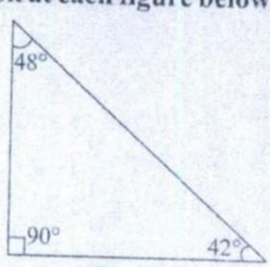


Fig. 6.9

(ii)

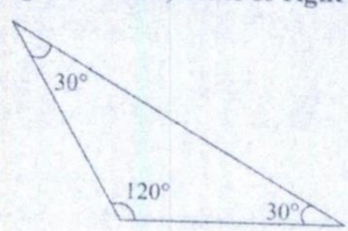


Fig. 6.10

(iii)

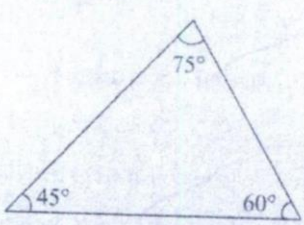


Fig. 6.11

(iv)

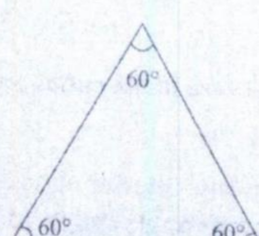


Fig. 6.12

3. Name all the three angles, sides and vertices of the triangle ABC in figure 6.13. Name the side opposite to vertex C.

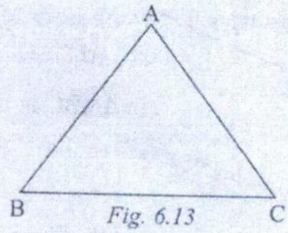


Fig. 6.13

4. In figure 6.14, identify the points which are

(i) in the exterior of the triangle;

(ii) in the interior of the triangle; and

(iii) on the triangle.

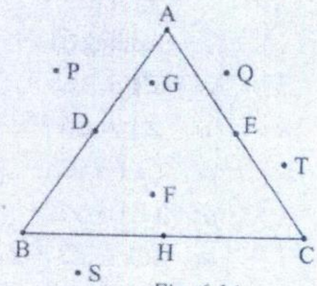


Fig. 6.14

5. Can a triangle have.

(i) more than one right angle?

(ii) a right angle and an acute angle?

(iii) an obtuse angle and a right angle?

(iv) an obtuse angle and an acute angle?

(v) more than one obtuse angle?

(vi) more than one acute angle?

(vii) two sides?

(viii) all angles more than 60°?

6. Name all the triangles in the figure 6.15 (i), (ii)

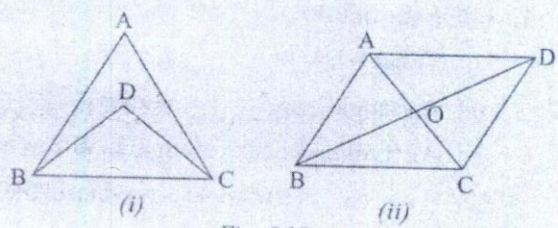


Fig. 6.15

7. Classify the triangles shown below as scalene, isosceles or equilateral. The length of the sides are given in the figures.

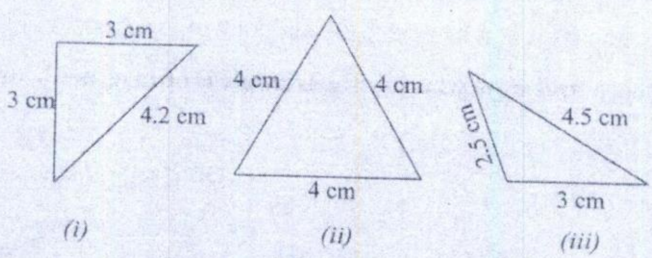


Fig. 6.16

8. Find the measure of the third angle when the two are given.

(i) 42°, 80°

(ii) 45°, 90°

(iii) 115°, 35°

(iv) 60°, 40°

(v) 74°, 42°

(vi) 50°, 110°

9. The angles of a triangle are in the ratios given below. Find the measure of each angle.

(i) 1 : 3 : 5

(ii) 4 : 5 : 9

(iii) 2 : 4 : 4

(iv) 1 : 1 : 1

10. The exterior angle and one interior opposite angle of each triangle are given here respectively. Find the other interior opposite angle and the third angle of each triangle.

(i) 120°, 45°

(ii) 95°, 35°

(iii) 58°, 20°

(iv) 130°, 60°

Exercise 6.2

1. **In figure 6.29, D is a point on the side AC of ΔABC.**

   Fill in the blanks with > or < or = :

   (i) BD .... AB + AD (ii) BC + CD......... BD

   (iii) BD .... $\frac{1}{2}$(AB + BC + AC)

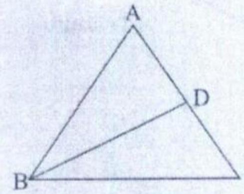


Fig. 6.29

2. In figure 6.30, P and Q are the points on the side BC of ΔABC. And AP = AQ.

   Prove that. AC + AB + BC > 2AP + PQ.

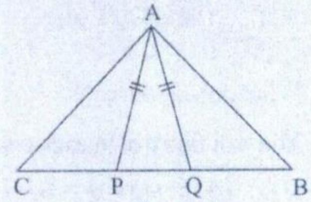


Fig. 6.30

3. **Which of the following can be the sides of a triangle?**

   (i) 1.8 cm, 3.5 cm, 6 cm (ii) 1 cm, 3 cm, 2 cm (iii) 1.5 cm, 2.5 cm, 5 cm

   (iv) 5 cm, 7 cm, 12 cm (v) 8.5 cm, 2 cm, 5 cm (vi) 3.4 cm, 2.1 cm, 5.3 cm

4. Three points A, B, C are collinear and B is the midpoint of A and C as shown in figure 6.31. Can you draw a triangle ABC?

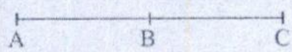


Fig. 6.31

5. **In figure 6.32, is**

   (i) OA + OB > AB?

   (ii) OA + OC = AC?

   (iii) OB + OC < BC?

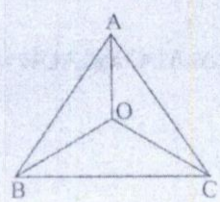


Fig. 6.32

Two Special Triangles: Equilateral and Isosceles

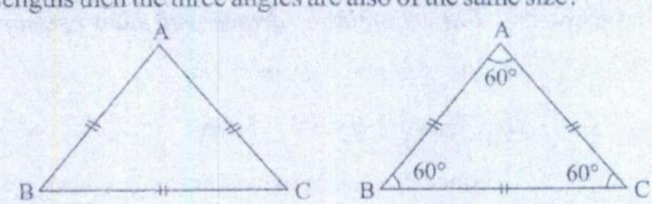
A triangle in which all the three sides are of equal lengths is called an equilateral triangle.

Take two copies of an equilateral triangle ABC as shown figure 6.33. Keep one of them fixed. Place the second triangle on it. It fits exactly into the first. Turn it round in any way and still they fit with one another exactly. Are you able to see that when the three sides of a triangle have equal lengths then the three angles are also of the same size?

We conclude that in an equilateral triangle:

(i) all sides have same length.

(ii) each angle has measure 60°.



(i) (ii)

Fig. 6.33

A triangle in which two sides are of equal lengths is called an isosceles triangle.

From a piece of paper cut out an isosceles triangel XYZ, with XY = XZ as shown in figure 6.34. Fold it such that Z lies on Y. The line XM through X is now the axis of symmetry. You find that ∠Y and ∠Z fit on each other exactly. XY and XZ are called equal sides; YZ is called the base; ∠Y and ∠Z are called base angles and these are also equal.

7. In figure 6.44, PQR is an isosceles triangle with PR = QR.

If $\angle P = 80°$, find

(i) $\angle PQR$ and $\angle PRQ$, and

(ii) the values of $\angle a$ and $\angle b$.

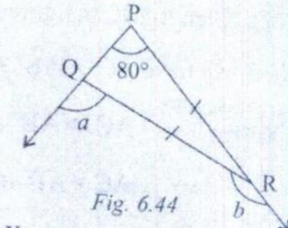
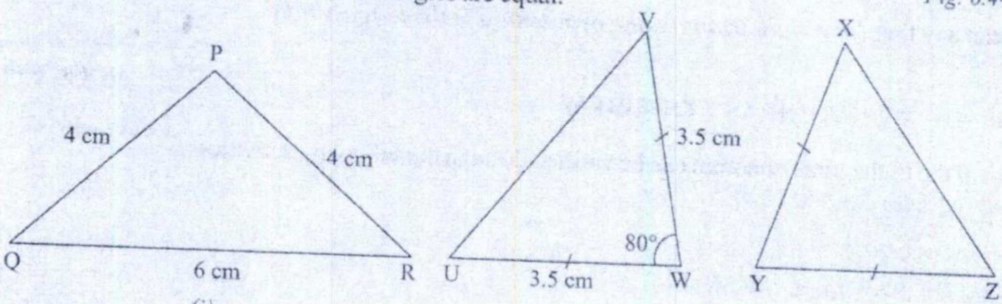


Fig. 6.44

8. In figures 6.45, identify which of the two angles are equal.



(i) *(ii)* *(iii)*

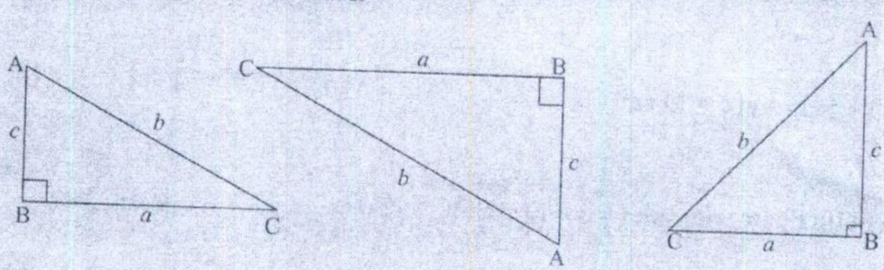
Fig. 6.45

Pythagoras' Theorem

To understand this concept, let us conduct an activity.

Activity 6

Draw any three right triangles as shown in Figure, each labelled as $\angle ABC$ with $\angle B = 90°$ in each case. Measure the sides a, c and the hypotenuse b in each case.



Calculate a^2, c^2 and b^2 and tabulate the observations as under:

Right Triangle	Measurements			Squares				Differences
	a	c	b	a^2	c^2	$a^2 + c^2$	b^2	$b^2 - (a^2 + c^2)$
(i)	3.8 cm	2 cm	4.3 cm	14.44 cm^2	4 cm^2	18.44 cm^2	18.49 cm^2	0.05 cm^2
(ii)								
(iii)								

What do we observe above? We observe that in each case the difference $b^2 - (a^2 + c^2)$ is either zero or so small that the same may be considered as zero.

$\therefore \quad b^2 - (a^2 + c^2) = 0 \quad \Rightarrow \quad b^2 = a^2 + c^2$

Thus, the above activity states that *'in a right triangle, the square of the hypotenuse equals the sum of the squares of its sides'*.

Hence, the relation between the lengths of the sides of a right triangle is known as Pythagoras' theorem.

$\therefore \quad \angle Y = \angle Z$ [$\because$ Angles opposite equal sides of a Δ are equal]

$\because \quad \angle Y = 50°$

$\therefore \quad \angle Z = 50°$

Using angle sum property of a triangle, we have $\angle X + \angle Y + \angle Z = 180°$

$\Rightarrow \quad \angle X + 50° + 50° = 180°$

$\Rightarrow \quad \angle X = 180° - (50° + 50°) = 180° - 100° = 80°$

Hence, $\mathbf{\angle X = 80°}$ and $\mathbf{\angle Z = 50°}$.

Exercise 6.3

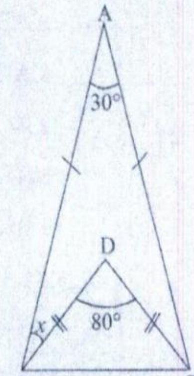


Fig. 6.39

1. In figure 6.39, ΔABC and ΔDBC are isosceles. Find

(i) $\angle ABC$ and $\angle ACB$ (ii) $\angle DBC$ and $\angle DCB$ (iii) $\angle x$

2. In ΔPQR, PQ = PR. Write its two equal angles.

3. In figures 6.40, identify which of the two sides are equal.

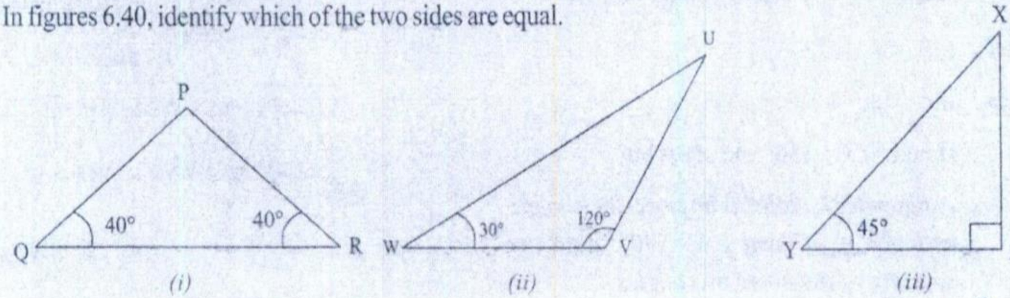


Fig. 6.40

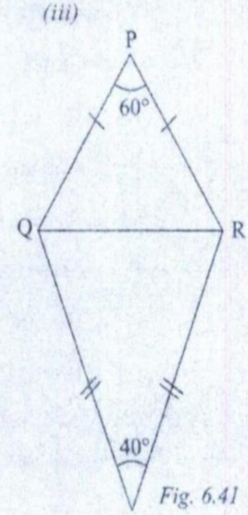


Fig. 6.41

4. In figure 6.41, ΔPQR and ΔSQR are both isosceles with common base QR. The equal sides have been shown with similar markings.

If $\angle P = 60°$ and $\angle S = 40°$, find

(i) $\angle PQR$ and $\angle PRQ$

(ii) $\angle SQR$ and $\angle SRQ$

(iii) $\angle PQS$ and $\angle PRS$.

Are these equal?

5. In figure 6.42, AC = CB and CD = BD.

Find the values of $\angle x$, $\angle y$ and $\angle z$.

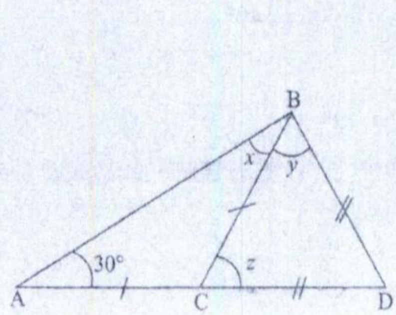


Fig. 6.42

6. In ΔXYZ in figure 6.43, $\angle X = \angle Z$. Write the two equal sides.

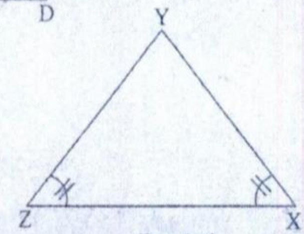


Fig. 6.43

Solved Examples

Example 1: *Two buildings 30 m and 15 m high, stand upright on a ground. If they are 36 m apart, find the distance between their tops.*

Let AB and CD represent the given buildings such that

AB = 30 m, CD = 15 m and AC = 36 m

Join BD. From D, draw DE ⊥ AB.

Then, ED = AC = 36 m

BE = AB – AE

= AB – CD = (30 – 15) m = 15 m

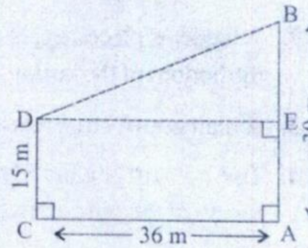


Fig. 6.47

Now, in right ΔBED in figure, by Pythagoras' theorem, we have

$BD^2 = BE^2 + ED^2$

$= 15^2 + 36^2 = 225 + 1296 = 1521 = 39^2$

So, BD = 39 m

Hence, the distance between the tops of the buildings is **39 m**.

Example 2: *An iron rod 10 m long is placed against a wall in such a way that the foot of the rod is 6 m away from the wall. Find how high the top of the iron rod reaches on the wall?*

Let AB represent the iron rod and A is the foot of the rod, which is 6 m away from the wall BC.

Then, clearly ABC in figure is a right angle triangle in which AB = 10 m and AC = 6 m.

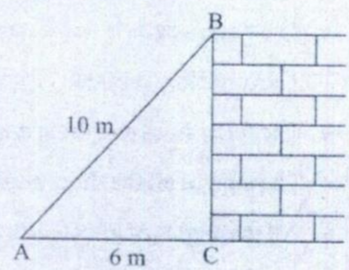
By Pythagoras' theorem, we have $AB^2 = AC^2 + BC^2$

⇒ $BC^2 = AB^2 - AC^2$

⇒ $BC^2 = 10^2 - 6^2 = 100 - 36$

⇒ $BC^2 = 64$

⇒ BC = 8 m



Hence, the height of the wall to which the top of the iron rod reaches is **8 m**.

Fig. 6.48

Example 3: *Find the length of the hypotenuse of a right triangle whose other two sides are 8 cm and 15 cm.*

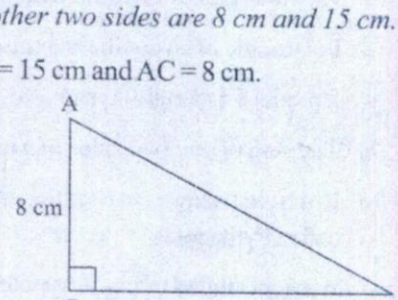
Let ABC in figure be a right triangle, right angled at C. Let BC = 15 cm and AC = 8 cm.

Then by Pythagoras' theorem, we have

$AB^2 = AC^2 + BC^2 = 8^2 + 15^2$

⇒ $AB^2 = 64 + 225 = 289$

⇒ AB = 17 cm



Hence, the required hypotenuse is **17 cm**.

Fig. 6.49

Exercise 6.4

1. **Verify that the following numbers represent Pythagorean triplet:**

   (i) 0.7, 2.4, 2.5 (ii) 27, 36, 45 (iii) 16, 63, 65

2. A ladder is placed against a wall of a building, 16 m above the ground. The foot of the ladder is 12 m away from the bottom of the building. What is the length of the ladder?

3. A man goes 6 km south and then 8 km east. Find how far away is he from his initial position?

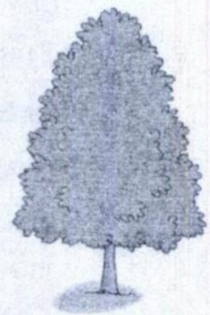
4. Two poles of heights 58 m and 10 m stand upright on the ground, the distance between them being 14 m. Find the length of the wire stretched from the top of one pole to the top of the other pole.

5. Find the length of the diagonal of a rectangle whose sides are 16 m and 30 m.

6. **In a right ΔABC, right angled at B, find AC, if**

   (i) AB = 10 cm, BC = 24 cm (ii) AB = 7 cm, BC = 24 cm

7. A tree is broken by the wind. If the point from where it broke is 12 m above the ground and its top touches the ground at a distance of 35 m from its foot, find out the total height of the tree before it broke.



8. If the two sides of right triangle are equal and the square of hypotenuse measures 128 cm^2, find the length of each side.

QUICK RECALL

- The three sided closed figure obtained by joining three non-collinear points is called a triangle.
- The three line segments forming a triangle are called the sides of the triangle.
- The three sides and three angles of a triangle together form the six elements of the triangle.
- The sum of all the three angles of a triangle is always equal to two right angles or 180°.
- All the angles of a scalene triangle are unequal.
- In an isosceles triangle, two angles are equal.
- All the angles of an equilateral triangle are equal.
- Each angle of an equilateral triangle is 60°.
- If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.
- The sum of any two sides of a triangle is greater than the third side.
- In a right triangle, the square of the hypotenuse equals the sum of the squares of the remaining two sides. It is called Pythagoras' theorem.
- In a right angled triangle, hypotenuse is the longest side.
- A set of three natural numbers a, b, c in this order forms a Pythagorean triplet if $c^2 = a^2 + b^2$.